Πάντα ῥεῖ ὡς ποταμός
(Everything flows like a river)

What has Eraclitus of Ephesus to do with fluid dynamics?

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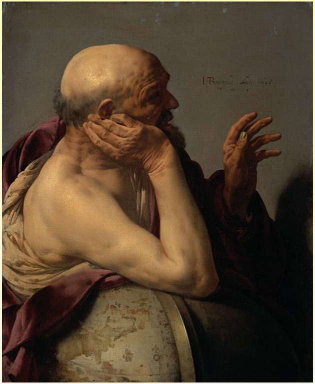
Yasoui, IRAN (September 2016)



Heraclitus de Ephesus (535 - 475 BCE)

(Hendrick ter Brugghen, 1628. Rijksmuseum, Amsterdam)

Fluid dynamics is a difficult subject!

Combining simplifications, approximations and the hope to gain a better understanding of the real physics lead sometimes to **paradoxes!**

Examples: some very well-known (Stokes, D'Alembert), others more subtle and less known (Whitehead, Olmstead & Gautesen, Sternberg & Koiter...)

Sometimes apparently unsolvable conflicts appear: this is the case for the classical **Bingham model** "marriage" with the so-called **lubrication approximation**

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Constitutive equation (2D geometry)

$\boldsymbol{u} = (u_1, u_2)$ (velocity)

$\mathbf{T}^* = -P^*\mathbf{I} + \mathbf{S}^*$ (Cauchy stress tensor)

$\mathbf{D}^*$ (rate of deformation tensor)

$$II_{\mathbf{S}^*} = \sqrt{\frac{1}{2}\,\mathrm{tr}\;(\mathbf{S}^{*2})},$$

$$II_{\mathbf{D}^*} = \sqrt{\frac{1}{2}\,\mathrm{tr}\;(\mathbf{D}^{*2})}$$ (second invariant of $\mathbf{S}^*$ and $\mathbf{D}^*$ respectively)

Constitutive equation (2D geometry)

Define $\Theta(z)$ is the **Heaviside stepwise function**

Tensor $\mathbf{S}^*$ is defined implicitely:

$$\left[\mathbf{S}^* - \left(2\eta^* + \frac{\boldsymbol{\tau}_o^*}{II_{\mathbf{D}^*}}\right)\mathbf{D}^*\right]\Theta\left(II_{\mathbf{S}^*} - \boldsymbol{\tau}_o^*\right) + \mathbf{D}^*\Theta\left(\boldsymbol{\tau}_o^* - II_{\mathbf{S}^*}\right) = 0, \tag{1}$$

η^* is the **plastic viscosity**,

$\boldsymbol{\tau}_o^*$ is the **yield stress**

When the shear stress is below $\boldsymbol{\tau}_o^*$ a solid-like behavior is observed, when it is above a fluid-like behavior appears.

The related free boundary problem

$$\varrho \frac{\mathrm{D}\ \boldsymbol{u}}{\mathrm{Dt}} = -\nabla p + \nabla \cdot \mathbf{S}^* \text{ (equation of motion)}$$

$$\mathrm{tr}\ \mathbf{D}^* = \frac{\partial u_1^*}{\partial x^*} + \frac{\partial u_2^*}{\partial y^*} = 0 \text{ (incompressibility condition)}$$

von Mises's criterion: transition between solid<->liquid regimes depends upon being $II_{\mathbf{S}^*} \leq \boldsymbol{\tau}_o^*$ (rigid) $\mathbf{D}^* = 0$) or $II_{\mathbf{S}^*} \geq \boldsymbol{\tau}_o^*$ (fluid $\mathbf{D}^* \neq 0$).

An unknown boundary separates these regimes, implicitely defined by $II_{\mathbf{S}^*} = \boldsymbol{\tau}_o^*$!

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Remarks

1. The Bingham model is very well known and used. Nevertheless *In some experiments it seems that the solid-like behavior is just a matter of the time scale used. That is* everything flows if you wait long enough *(Heraclitus)*.
   In some experiments it seems that there is no evidence of a yield stress at very low shear rate. But "very low" is a relative concept highly dependent on the experimental devices..
2. Academic debate: is the yield stress a real property of some materials?

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¡At day-long time scale it appears to be **rigid**!

At several year long time scale it appears to be **liquid**!

Notice: the pitch viscosity is about 100 billones (10^{11}) times water viscosity! How long one has to wait to see a single drop?

The pitch drop experiment

Cronology (Queensland University, AU)

date	detach event	time interval
1927	setup	
1930	cut of the neck	
Dic. 1938	first drop	8.9
Feb. 1947	second drop	8.3
Abr. 1954	third drop	7.2
May 1962	fourth drop	8.1
Aug. 1970	fifth drop	8.3
Abr. 1979	sixth drop	8.7
Jul. 1988	seventh drop	9.3
Nov. 2000	eighth drop	12.3
Apr. 2014 ?	nineth drop	14

The debate

Barnes (J. Non-Newtonian Fluid Mech., 1999) *The yield stress – a review of “ Πάντα ῥεῖ” – Everything flows?*

“These materials nevertheless show slow but continual steady deformation when stressed for a long time below yield stress, having shown an initial linear elastic response to the applied stress”. The existence of a yield stress is a myth!

Astarita (J. Rheol., 1990) *The engineering reality of the yield stress*

“Whether yield stress is or is not an engineering reality depends on what problem we are considering”